Register Number	Class	Name

PEICAI SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2010
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

MATHEMATICS (4016/01)
PAPER 1

DATE: 3 SEPTEMBER 2010 (FRIDAY)
DURATION: 2 HOURS

READ THESE INSTRUCTIONS FIRST

Write your Register number, class and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use

This document consists of **16** printed pages and **0** blank page.

Set by: Ms June Yeo
Vetted by: Ms Lee Chiou Kwei

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

For Examiner's Use

1 **(a)** Given that $ax - by = 2 - 3x$, find an expression for x in terms of a, b and y.

(b) Simplify $\dfrac{2x^2 - x - 3}{(2x-3)(3+x)}$.

Answer: (a) _____ [1]

(b) _____ [2]

2 y is inversely proportional to $(x-3)$.

(a) Write down an expression for y in terms of x and a constant k.

(b) It is given that $y = 6$ when $x = 5$. Find y when $x = 11$.

Answer: (a) _____ [1]

(b) y = _____ [2]

For Examiner's Use

For Examiner's Use

3

(a) (i) Simplify $\frac{6ab}{c^2} \div \frac{2a^2}{bc} \times \frac{a^3b}{9c}$.

(ii) Solve the equation $6^{3x+2} = 1296$.

(b) Find the difference between the Highest Common Factor (HCF) and Lowest Common Multiple (LCM) of 30 and 42.

For Examiner's Use

Answer: (a)(i) _____ [1]

(ii) x = _____ [2]

(b) _____ [2]

For Examiner's Use

4 **(a)** Solve the simultaneous equations

$$2x - 3y = 19$$
$$3x + 2y = -4$$

(b) Solve the equation $\frac{3}{p} = \frac{4p}{27}$.

Answer: (a) $x =$ _____, $y =$ _____ [3]

(b) $p =$ _____ [2]

For Examiner's Use

For Examiner's Use

5 A craftsman makes a model of a house to a scale of 1 : 10.

(a) Given that the height of the roof on the model is 86 cm, find the height of the actual roof, giving your answer in metres.

(b) Given that the actual area of the kitchen floor is 20 m^2, find, in square metres, the area of the kitchen floor on the model.

Answer: (a) _____ m [1]

(b) _____ m^2 [2]

For Examiner's Use

6 **(a)** Find the number of sides of a polygon in which the sum of the interior angles is four times the sum of the exterior angles.

(b) A senior citizens' club had 98 members and the ratio of men to women is 5 : 9. After 22 new members join the club, the ratio of men to women became 9 : 11. How many men joined the club?

Answer: (a) _____ sides [2]

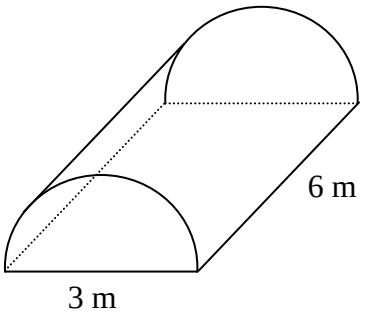
(b) _____ men [2]

For Examiner's Use

7 The diagram shows a closed container in the shape of half a cylinder. Its diameter is 3 m and its length is 6 m.

(a) Find the total external surface area of the container.

(b) The exterior of the container is to be painted. If painting costs \$2.80 per m^2, find the cost of painting the container to the nearest dollar.



Answer: (a) _____ m^2 [2]

(b) \$_____ [1]

For Examiner's Use

8 **(a)** Solve the inequalities for integer values of x.

$$3x-2 \le 4x-8 < 3x+2$$

(b) Factorise completely $3pr + 3ps - r - s$.

Answer: (a) _____ [2]

(b) _____ [2]

For Examiner's Use

9 A large pipe has a radius of 7.2×10^2 mm. A small tube has a diameter of 16×10^{-3} mm. Calculate

(a) the radius of the small tube,

(b) the ratio of the radius of large pipe to the small tube,

giving your answers in standard form.

Answer: (a) _____ mm [1]

(b) _____ [1]

10 **(a)** The dimensions of a rectangle are 25 cm by 20 cm. The length of the rectangle is increased by 10% while its width is decreased by 20%. Calculate the percentage change in area.

(b) Mr Tan invested $6 000 in a bank at 5% simple interest per annum. After 3 years, he withdraws all the money. How much money does he receive?

Answer: (a) _____% [2]

(b) $_____ [2]

For Examiner's Use

For Examiner's Use

11 The temperature at the top of Mount X was –20 ℃. At the same time, the temperature at the bottom of the mountain was 5 ℃.

(a) Calculate the difference between the two temperatures.

(b) The height of the mountain is 3600 m. Given that the temperature changed uniformly with height, calculate the height at which the temperature was 0 ℃.

Answer: (a) _____ ℃ [1]

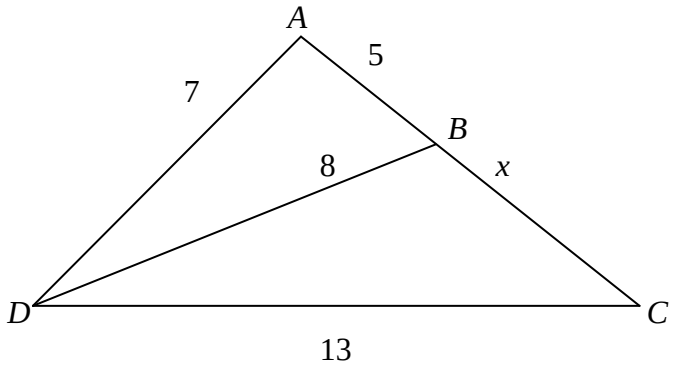
(b) _____ m [1]

12 In the diagram, ABC is a straight line. Given that AB = 5 cm, AD = 7 cm, CD = 13 cm, BD = 8 cm and $BC = x$ cm.

(a) Calculate $\cos \angle ABD$.

(b) (i) State the value of $\cos \angle CBD$.

(ii) Hence, find the value of x.



Answer: (a) $\cos \angle ABD$ = _____ [2]

(b)(i) $\cos \angle CBD$ = _____ [1]

(ii) x = _____ [3]

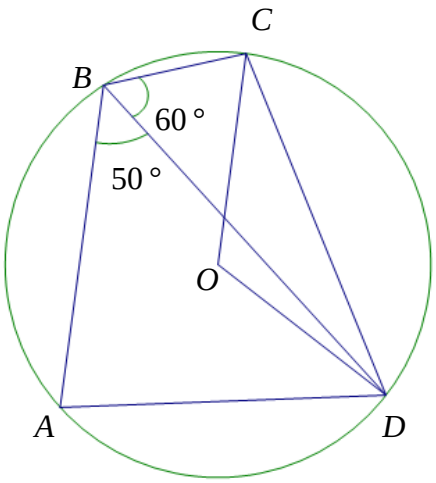
For Examiner's Use

13 The points A, B, C and D lie on a circle centre O. $\hat{ABD} = 50°$ and $\hat{DBC} = 60°$. Calculate

(a) $\hat{COD}$,

(b) $\hat{ADC}$,

(c) $\hat{ADO}$.



For Examiner's Use

Answer:

(a) $\hat{COD}$ = _____ °[1]

(b) $\hat{ADC}$ = _____ °[1]

(c) $\hat{ADO}$ = _____ °[1]

For Examiner's Use

14 A bag contains a number of balls each one coloured either red or blue. A ball is chosen at random and then put back into the bag. This process is repeated several times.

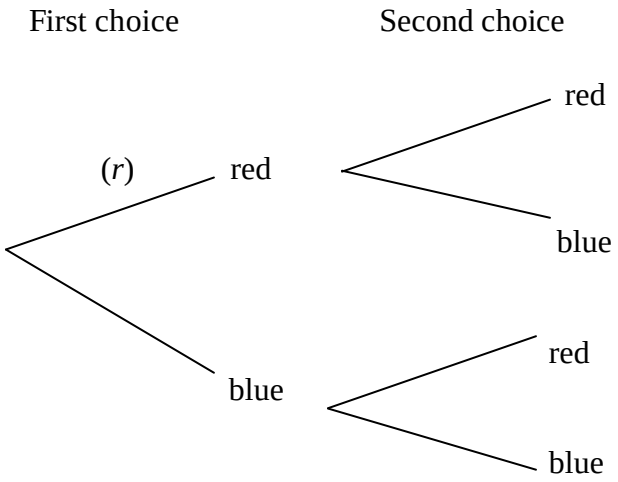
(a) The probability of choosing a red ball is r. Write down, in terms of r, the probability of choosing a blue ball.

(b) The tree diagram below represents the situation when the process has been carried out twice. Expressing your answer in terms of r, find the probability that a red ball was chosen each time.

(c) The process was carried out nine times. Find the probability that

(i) a red ball was chosen every time,

(ii) at least one blue ball was chosen.



Answer: (a) _____ [1]

(b) _____ [1]

(c)(i) _____ [1]

(ii) _____ [1]

For Examiner's Use

For Examiner's Use

15 **(a)** Draw accurately the triangle ABC with $\angle CAB = 40°$ and $AC = 10$ cm. The base $AB = 12$ cm has been drawn for you. Measure, and write down, the length of BC.

(b) Draw the perpendicular bisector of BC. [1]

Answer (a) & (b)

[1]

Answer: BC = _____ cm [1]

For Examiner's Use

For Examiner's Use

16 **(a)** An ice cream stall sells both strawberry and chocolate ice cream. A small portion costs 40 cents and a large portion costs 60 cents. During a short period of time, the number of ice creams sold is shown in the table below.

	small	large
strawberry	2	3
chocolate	5	4

Given that $\mathbf{P} = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 40 \\ 60 \end{pmatrix}$

(i) find **PQ**,

(ii) explain what the numbers given in your answer to part **(a)** represent.

(b) Two cuboids are geometrically similar. The ratio of the surface areas of the bigger cuboid to the smaller cuboid is $5b : 3b - 5$ and the ratio of the volumes of the cuboids is $\frac{8}{125}$. Find the value of b.

Answer: (a)(i) **PQ =** _____ [1]

(ii)

_____

_____ [1]

(b) b = _____ [3]

For Examiner's Use

17 P is the point (3, 4) and Q is the point (11, 10).

(a) Calculate the coordinates of the midpoint of PQ.

(b) Express $\overrightarrow{PQ}$ as a column vector.

(c) Find the value of $\left|\overrightarrow{PQ}\right|$.

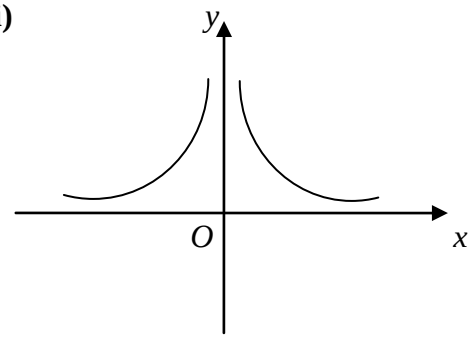
Answer: (a) (_____, _____) [1]

(b) $\overrightarrow{PQ}$ = _____ [1]

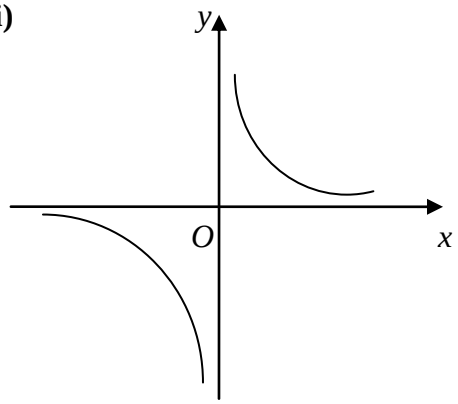
(c) $\left|\overrightarrow{PQ}\right|$ = _____ [1]

18 **(a)** The sketch of the graphs $y = x^n$ are shown below. Write down a possible value of n for each of the graphs.

(i)



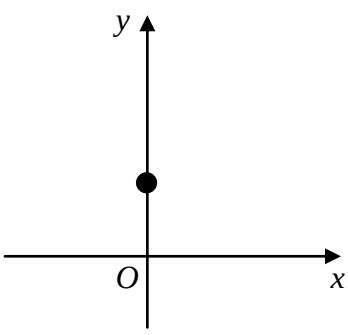
(ii)



Answer: (a)(i) n = _____ [1]

(ii) n = _____ [1]

(b) The point (0, 1) is marked on the diagram in the answer space. In the answer space, sketch the graph of $y = 1 - x$.



[1]

For Examiner's Use

19 In a contest, the time taken, in minutes, by 15 students to send a similar SMS message on their handphone are recorded below.

2.0	1.9	3.6	2.7	2.1
3.6	0.5	4.5	0.8	2.0
3.8	2.1	1.3	2.1	1.2

(a) Draw a stem and leaf diagram to represent the above data.

(b) Calculate

(i) the lower quartile,

(ii) the median and

(iii) the upper quartile

of this distribution.

(c) Hence, draw a box-and-whisker plot.

(d) How long did the winner of the contest take to send the SMS message?

Answer (a)(i)

[2]

Answer (c)

[2]

Answer: (b)(i) _____ minutes [1]

(ii) _____ minutes [1]

(iii) _____ minutes [1]

(d) _____ minutes [1]

For Examiner's Use

20 The diagram is the speed-time graph for a particular journey.

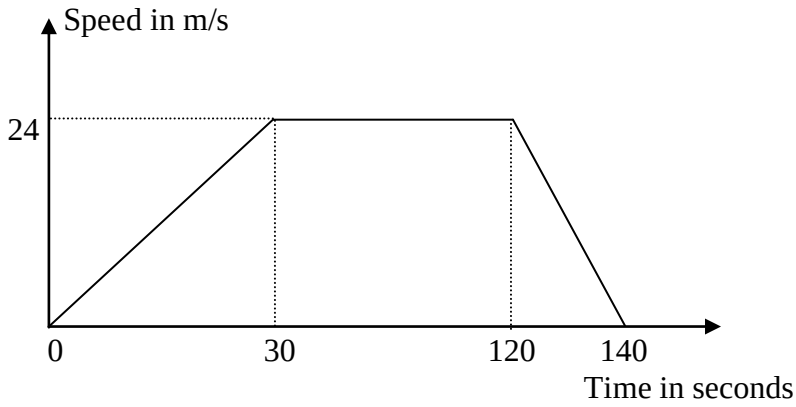
For Examiner's Use

(a) calculate

(i) the retardation during the last 20 seconds,

(ii) the speed after 25 seconds,

(iii) the total distance travelled.



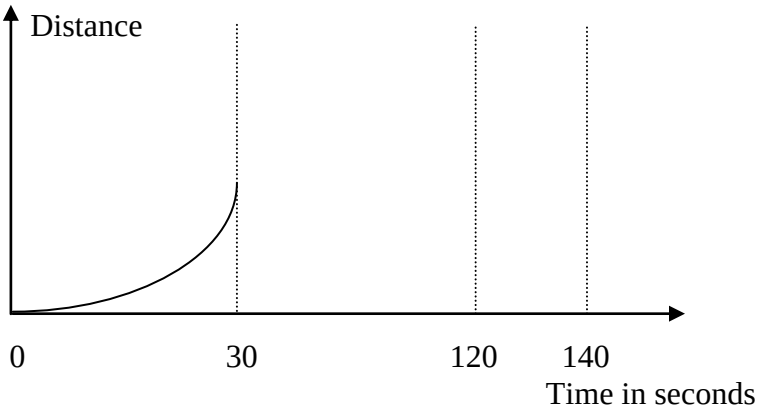
Answer: (a)(i) retardation = _____ m/s^2 [1]

(ii) speed = _____ m/s [2]

(iii) total distance = _____ m [2]

(b) On the axes in the answer space, complete the sketch of the distance-time graph for the same journey.

Answer (b)



[2]

~ End of Paper ~

Peicai Secondary School
Math Department
Secondary 4 Express / 5 Normal Academic
Mathematics Paper 1 – Mark Scheme

1. (a)
$ax - by = 2 - 3x$
$ax + 3x = 2 + by$
$x(a+3) = 2 + by$
$x = \dfrac{2+by}{a+3}$ [A1]

(b)
$\dfrac{2x^2 - x - 3}{(2x-3)(3+x)}$
$= \dfrac{(2x-3)(x+1)}{(2x-3)(3+x)}$ [M1]
$= \dfrac{x+1}{3+x}$ [A1]

2. (a)
$y = \dfrac{k}{(x-3)}$ [A1]

(b) $y = 6$, $x = 5$: $6 = \dfrac{k}{(5-3)}$
$k = 12$ [M1]
$y = \dfrac{12}{(x-3)}$
$x = 11$:
$y = \dfrac{12}{11-3} = 1.5$ [A1]

3. (a) (i)
$\dfrac{6ab}{c^2} \div \dfrac{2a^2}{bc} \times \dfrac{a^3b}{9c}$
$= \dfrac{6ab}{c^2} \times \dfrac{bc}{2a^2} \times \dfrac{a^3b}{9c}$
$= \dfrac{b}{c^2} \times \dfrac{b}{1} \times \dfrac{a^2b}{3}$
$= \dfrac{a^2b^3}{3c^2}$ [A1]

(ii)
$6^{3x+2} = 1296$
$6^{3x+2} = 6^4$
$3x + 2 = 4$ [M1]
$3x = 2$
$x = \dfrac{2}{3}$ [A1]

(b)

2	30	42
3	15	21
	5	7

HCF = $2 \times 3 = 6$

2	30	42
3	15	21
5	5	7
7	1	7
	1	1

LCM = $2 \times 3 \times 5 \times 7 = 210$
HCF = 6; LCM = 210 [M1]
difference = 210 – 6 = 204 [A1]

4. (a)
$2x - 3y = 19 - - - (1)$
$3x + 2y = -4 - - - (2)$
$(1) \times 3: \ 6x - 9y = 57 - - - (3)$
$(2) \times 2: \ 6x + 4y = -8 - - - (4)$
$(3) - (4): \ -13y = 65$ [$M1$]
$y = -5$ [$A1$]
$y = -5: \ 3x + 2(-5) = -4$
$3x = 6$
$x = 2$ [$A1$]

(b)
$\frac{3}{p} = \frac{4p}{27}$
$4p^2 = 81$ [$M1$]
$p^2 = \frac{81}{4}$
$p = \pm\frac{9}{2}$ [$A1$]

5. (a) 1 cm --- 10 cm
86 cm --- 860 cm = 8.6 m [A1]

(b) 0.1 m --- 1 cm
0.01 m^2 --- 1 cm^2
20 m^2 --- $\frac{20}{0.01}$ [M1] = 2000 cm^2 = 0.2 m^2 [A1]

6. (a) $180°(n - 2) = 4 \times 360°$ [M1]
n = 10 [A1]

(b) before:
14 units --- 98 members
1 unit --- 7 members
5 units --- 35 men
after:
20 units --- 120 members
1 unit --- 6 members
9 units --- 54 members
above steps: [M1]
54 – 35 = 19 men [A1]

7. (a) total external surface area
$= (3 \times 6) + \left(\pi \times \left(\frac{3}{2}\right)^2\right) + \frac{1}{2}\left(2 \times \pi \times \frac{3}{2} \times 6\right)$ [$M2$]
$= 53.3 \ m^2 \ (3 \ significant \ figures)$ [$A1$]

(b) 1 m^2 --- \$2.80
53.3 m^2 --- \$149 (nearest dollar) [A1]

$3x-2\le 4x-8<3x+2$

$3x-2\le 4x-8$

8. (a) $6\le x$ [M1]

$4x-8<3x+2$

$x<10$

$6\le x<10$

x = 6, 7, 8 & 9 [A1]

(b)

$3pr+3ps-r-s$

$=3p(r+s)-r-s$

$=3p(r+s)-(r+s)$ [$M1$]

$=(r+s)(3p-1)$ [$A1$]

9. (a) *radius*

$=\dfrac{16\times10^{-3}}{2}$

$=8\times10^{-3}$ *mm* [$A1$]

(b) $\dfrac{7.2\times10^{2}}{8\times10^{-3}}$ [$M1$]

$=9\times10^{4}$ [$A1$]

10. (a) $\dfrac{(1.1\times25\times0.8\times20)-(25\times20)}{25\times20}\times100\%$ [$M1$]

$=12\%$ [$A1$]

(b) $\dfrac{(6000)(5)(3)}{100}+6000$ [$M1$]

$=\$6900$ [$A1$]

11. (a) $5-(-20)=25$ ℃ [A1]

(b) $25°C---3600\ m$

$1°C---144\ m$

$5\times144=720\ m$ [$A1$]

12. (a) $\cos A\hat{B}D$

$=\dfrac{5^2+8^2-7^2}{2(5)(8)}$ [$M1$]

$=\dfrac{1}{2}$ [$A1$]

(b)(i) $\cos C\hat{B}D=-\dfrac{1}{2}$

(b)(ii)

$$\frac{x^2+8^2-13^2}{2(x)(8)}=-\frac{1}{2}\ [M1]$$

$$-8x=x^2-105$$

$$x^2+8x-105=0$$

$$(x+15)(x-7)=0$$

$x=-15\ (rejected),\ 7\ [A1]$

13. (a) $\angle COD=120^\circ\ (\angle\ @\ centre\ =\ 2\angle\ @\ circumference)\ [A1]$

(b)

$$\angle ADC=180^\circ-50^\circ-60^\circ\ (opp\ \angle s\ of\ cyclic\ quad.)$$

$$\angle ADC=70^\circ\ [A1]$$

(c)

$$\angle CDO=\frac{180^\circ-120^\circ}{2}\ (\angle\ sum\ of\ \Delta)$$

$$=30^\circ\ [M1]$$

$$\angle ADO=70^\circ-30^\circ$$

$$=40^\circ\ [A1]$$

14. (a) P(blue ball) = $(1 - r)$ [A1]

(b) P(each time – red ball) = r^2 [A1]

(c) (i) P(red ball – 9 times) = r^9 [A1]

(ii) P(at least 1 blue ball) = $(1 - r^9)$[A1]

16. (a) (i)

$$PQ=\begin{pmatrix}2 & 3\\ 5 & 4\end{pmatrix}\begin{pmatrix}40\\ 60\end{pmatrix}$$

$$=(80+180\quad 200+240)$$

$$=(260\quad 440)\ [A1]$$

(ii) element 260 cents tells the total sales of 2 small & 3 large strawberry portions whereas element 480 cents gives the total sales of 5 small and 4 large chocolate portions [A1]

$$\frac{3b-5}{5b} = \left(\sqrt[3]{\frac{8}{125}}\right)^2 \quad [M2]$$

$$\frac{3b-5}{5b} = \left(\frac{2}{5}\right)^2$$

$$\frac{3b-5}{5b} = \frac{4}{25}$$

(b) $20b = 75b - 125$

$55b = 125$

$b = 2\frac{3}{11} / 2.27 \ \left(3 \ \textit{significant figures}\right) \ [A1]$

note: *accept either* 1 *above value for b*

17. (a)

$$M_{PQ} = \left(\frac{3+11}{2}, \ \frac{4+10}{2}\right)$$

$$= \left(7, \ 7\right) \ [A1]$$

(b)

$$\overrightarrow{PQ} = -\begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}11\\10\end{pmatrix}$$

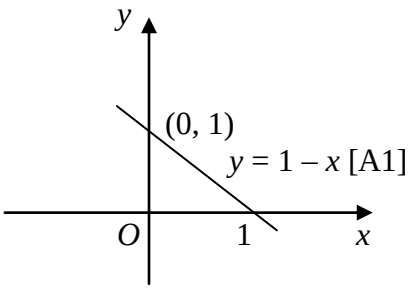
$$= \begin{pmatrix}8\\6\end{pmatrix} \ [A1]$$

(c) $\left|\overrightarrow{PQ}\right| = \sqrt{6^2 + 8^2} = 10 \ \textit{units} \ [A1]$

18. (a) (i) $n = -2$ [A1]

(ii) $n = -1$ [A1]

(b)



19. (a)

0	5	8				
1	2	3	9			
2	0	0	1	1	1	7
3	6	6	8			
4	5					

Key: 0|5 means 0.5

[A2: 1 mark for (ordered) stem & leaf diagram, 1 mark for key]

(b) (i) 1.3 minutes [A1]

(ii) 2.1 minutes [A1]

(iii) 3.6 minutes [A1]

(d) 0.5 minute [A1]

20. (a) (i) retardation = $\frac{24}{20} = 1.2\ m/s^2$ [*A*1]

(ii) $\frac{v_{25}}{24} = \frac{25}{30}$ [*M*1]

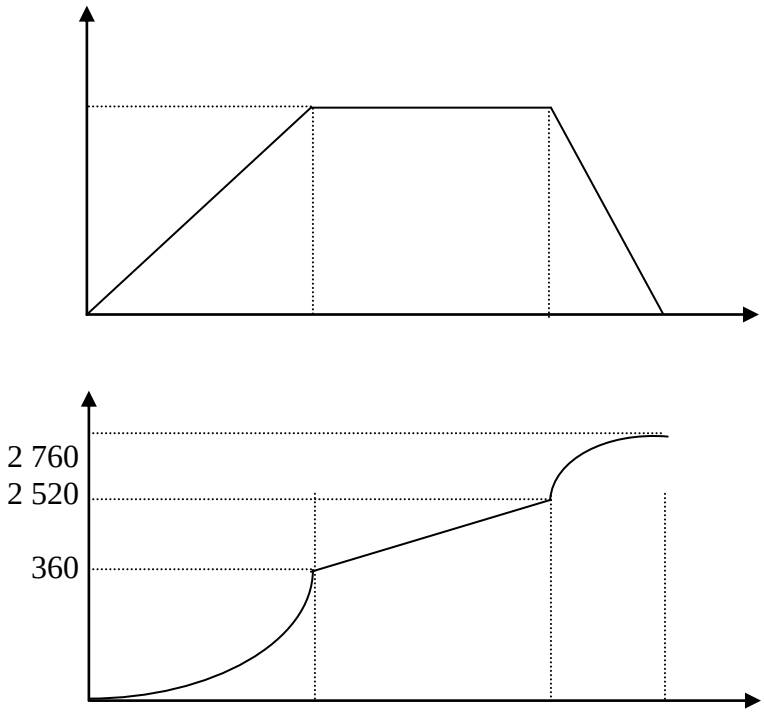
$v_{25} = 20\ m/s$ [*A*1]

(iii) total distance

$= \frac{1}{2}(120 - 30 + 140)(24)$ [*M*1]

$= 2760\ m$ [*A*1]

(b)



[A2]